1 Logarithms

1.1 Properties of Logarithms

$$\log_a b^n = n \log_a b$$

$$\log_a b + \log_a c = \log_a bc$$

$$\log_a b - \log_a c = \log_a \frac{b}{c}$$

$$\frac{\log_a b}{\log_a c} = \log_c b$$

$$\log_{a^n} b^n = log_a b$$

$$(\log_a b)(\log_b c) = \log_a c$$

$$(\log_a b)(\log_c d) = (\log_a d)(\log_c d)$$

Trick: $\lceil \log_{10} x \rceil$ gives you the number of digits of x

Trick: For problems that take the form $\log_a b = \log_c d$ and ask you to evaluate something like $\frac{b}{a}$, simply set both equations equal to some constant t and then set up a quadratic. For instance, when asked to find $\frac{q}{p}$:

$$\log_9 p = \log_{12} q = \log_{16}(p+q)$$

$$9^t = p \;\Big|\; 12^t = q \;\Big|\; 16^t = p+q$$

$$\frac{q}{p} = \left(\frac{12}{9}\right)^t = \left(\frac{4}{3}\right)^t$$

$$16^t = p + q = 9^t + 12^t$$

$$\frac{16^t}{12^t} = \left(\frac{4}{3}\right)^t = \frac{9^t + 12^t}{12^t} = \left(\frac{3}{4}\right)^t + 1$$

Let $x = \left(\frac{4}{3}\right)^t$:

$$x = \frac{1}{x} + 1$$

$$x^2 = 1 + x \implies x^2 - x - 1 = 0$$

$$x = \boxed{1 + \sqrt{2}}$$

2 Not Just for Right Triangles

Period is the amount of the graph we can draw before we must start repeating, while **frequency** is how often the graph of the function repeats. Let T represent the period and f represent the frequency:

$$f = \frac{1}{T}$$

Phase shift of a function $f(x-k)$ is the amount of the graph shifted from the parent function $f(x)$. **Amplitude** is half the difference between the largest and smallest values of a graph.

Trick: When asked to find the amplitude of $f(x) = a\sin x + b\cos x$, use the formula:

$$A = \sqrt{a^2 + b^2}$$

Trick: To find the period of $f(x) = \sin(\frac{ax}{b}) + \cos(\frac{cx}{d})$, where a, b, c, and d are integers and $\frac{a}{b}$ and $\frac{c}{d}$ are in lowest terms, use:

$$z = \frac{2\pi \cdot lcm(b,d)}{gcd(a,c)}$$

2.1 Trig Identities

- $\sin(a+b) = \sin a \cos b + \sin b \cos a$
- $\sin(a-b) = \sin a \cos b - \sin b \cos a$
- $\cos(a+b) = \cos a \cos b - \sin a \sin b$
- $\cos(a-b) = \cos a \cos b + \sin a \sin b$
- $\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$
- $\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$
- $\sin 2x = 2\sin x \cos x$
- $\cos 2x = \cos^2 x - \sin^2 x$
- $\cos 2x = 1 - 2\sin^2 x$
- $\cos 2x = 2\cos^2 x - 1$
- $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
- $1 + \tan^2 x = \sec^2 x$
- $1 + \cot^2 x = \csc^2 x$
- $\sin^2 x + \cos^2 x = 1$
- $\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$
- $\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$
- $\sin a - \sin b = 2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$

2.2 Common problem solving techniques

- Look for angles whose sum or difference is a multiple of 90°. If these exist, we can often use our relations like $\sin(180 - x) = \sin x$ and $\sin(90° - x) = \cos x$.
- When you see squares of trigonometric relations, try using $\sin^2 x + \cos^2 x = 1$ or the related identities
- Don't work with inverse trig functions. Apply trigonometric functions to equations involving inverse trig functions to get rid of them

2.3 Common factorizations

- $(\cos x - \sin x)^2 = 1 - \sin(2x)$
- $(\cos x + \sin x)^2 = 1 + \sin(2x)$

3 More Triangles!

3.1 Triangle Laws

- Law of Cosines: $c^2 = a^2 + b^2 - 2ab\cos C$
- Law of Sines: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$ where R is the circumradius of $\triangle ABC$.
- Law of Tangents: $\frac{a-b}{a+b} = \frac{\tan\frac{A-B}{2}}{\tan\frac{A+B}{2}}$
- Thales's Triangle Theorem: If a triangle has, as one side, the diameter of a circle, and the third vertex of the triangle is any point on the circumference of the circle, then the triangle will always be a right triangle.

3.2 Problem Recognition

In problems, you won't always be given a nice recognizable version of the Law of Cosines. Sometimes, you will be given expressions like:

$$\frac{c^2 - a^2 - b^2}{-2ab}$$

and you have to recognize that this is equal to $\cos C$. Also, in cases where you have the sines of angles and the circumradius of a triangle, you will likely have to use Law of Sines.
The presence of the circumradius or $\frac{a}{\sin A}$ terms is often a giveaway that the law of sines will be useful.

3.3 Areas

Area of $[ABC]$ is:

- $\frac{ah_a}{2}$
- $\frac{ab}{2}\sin C$
- $\frac{abc}{4R} \implies$ derived from previous formula using $\frac{c}{\sin C} = 2R$
- rs
- $\sqrt{s(s-a)(s-b)(s-c)}$

where r is the inradius, s is the semiperimeter, and h_a is the altitude to side BC